DETAILS EXPLANATIONS

[PART : A]

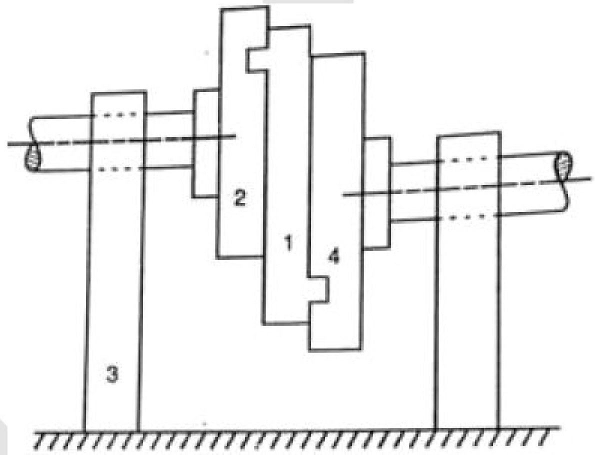
1. ***Lower Pair :***

When two links have surface or area contact between them while in motion, such a pair is known as lower pair

Higher Pair :

When two links have point or line contact between them while in motion, the pair so formed is termed as higher pair.

2. The third inversion of double slider crank chain is Oldham's coupling which is used to connect two parallel shafts whose axes are not in perfect alignment. In this inversion, the coupler link 3 is held fixed and the links 2 and 4 have slotted grooves to form sliding pairs with link 1



3. The term degree of freedom of a mechanism (dof) refers to number of independent input parameters which are required to specify the relative position of all the links.

4. ***(i) Referring to Figure (a)***

Number of links, $n = 7$

Number of lower pairs, $j = 8$

$$\text{dof} = 3(n - 1) - 2j$$
$$= 3(7 - 1) - 2 \times 8 = 2$$

(ii) Referring to Figure (b) in which the roller having slipping motion form a

Number of links, $n = 4$

Number of lower pairs, $j = 3$

Number of lower pairs, $h = 1$

$$\text{dof} = 3(n - 1) - 2j - h$$
$$= 3(4 - 1) - 2 \times 3 - 1 = 2$$

5. The coefficient of fluctuation is the permissible variation in speed and is defined as

$$C = \frac{\omega_1 - \omega_2}{\omega}$$

Where ω_1 = maximum angular sped of flywheel

ω_2 = minimum angular speed of flywheel

ω = average angular speed of flywheel

Also $$C = V_1 - \frac{V_2}{V}$$

Where V_1 = maximum speed of given point on flywheel.

V_2 = minimum speed of same point on flywheel.

V = average speed of same point on flywheel.

6. Presence of friction between pulley and belt causes differential tension in the belt. This differential tension causes the belt the elongate or contract and create a relative motion between the belt and the pulley surface. This relative motion between the belt and the pulley surface is created due to the phenomena known as elastic creep.

7. ***Circular Pitch :***

It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by pc.

Mathematically Circular pitch, $$p_c = \pi\frac{D}{T}$$

Where D = Diameter of the pitch circle

and T = Number of teeth on the wheel

A little consideration will show that the two gears will mesh together correctly, if the two wheel have the same circular pitch.

Diametral Pitch :

It is the ratio of number of teeth to the pitch circle diameter in millimeters. It is denoted by pd. Mathematically,

Diametral pitch $$p_d = \frac{T}{D} = \frac{\pi}{p_c} \qquad \left(\because p_c = \frac{\pi D}{T}\right)$$

Where T = Number of teeth

and D = Pitch circle diameter.

Module :

It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m. Mathematically

$$m = \frac{D}{T}$$

8. The contact ratio or the number of pairs of teeth in contact is defined as the ratio of the length of the arc of contact to the circular pitch contact ratio or number of pairs of teeth in contact

$$= \frac{\text{Length of the arc of contact}}{p_c}$$

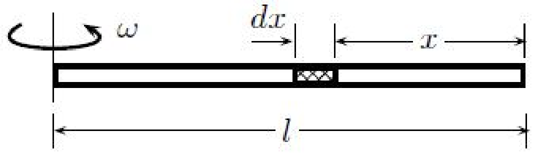
9. For angular displacement of θ in anticlockwise direction, the equilibrium equation is

$$300\theta \times 0.150 = k \times 0.3\theta \times 0.3$$
$$k = 300 \times 0.150/0.3 \times 0.3$$
$$= 500 \text{ N/m}$$

10. $$\frac{1}{2}kx^2 = mgh$$

$$x = \sqrt{\frac{2mgh}{k}} = 0.6324 \text{ m}$$

11. The condition presented in the problem is depicted in the following figure.



The extension of a small element of length dx at distance x from the free end is given by

$$\delta = \int_0^l \frac{1}{AE}\left(l - \frac{x}{2}\right)\omega^2 \frac{w}{g} x\,dx$$
$$= \frac{w\omega^2}{gAE}\left[l\frac{x^2}{2} - \frac{x^3}{6}\right]_0^l$$
$$= \frac{w\omega^2 l^3}{gAE}\left(\frac{1}{2} - \frac{1}{6}\right)$$
$$= \frac{w\omega^2 l^3}{3gAE}$$

12. Given that

$$\sigma_h = 100 \text{ MN/m}^2$$
$$\sigma_l = 50 \text{ MN/m}^2$$
$$E = 200 \text{ GN/m}^2$$
$$\mu = 0.3$$

Therefore,

$$\varepsilon_h = \frac{\sigma_h}{E}(1 - 2\mu)$$
$$= 0.425 \times 10^{-3}$$

13. Stiffness k is determined as

$$k = \frac{Gd^4}{64R^3 n}$$

Therefore,

$$\frac{k'}{k} = \frac{75^3 \times 10}{50^3 \times 15}$$
$$k' = 2.25k$$

Percentage increase in stiffness is

$$\% \text{ Increase} = \frac{2.25 - 1}{1} \times 100$$
$$= 125\%$$

14. Torsional stiffness is torque divided by angle of twist. Torsional stiffness in the three segments, S_1, S_2 and S_3, is given as

$$T_{S1} = 20 \text{ Nm/rad}$$
$$T_{S2} = 30 \text{ Nm/rad}$$
$$T_{S3} = 30 \text{ Nm/rad}$$
$$T = 10 \text{ Nm}$$

The net angular deflection (θ) will be sum of the angular deflection in the three segments, which are applied with the same amount of torque, T. Hence,

$$\theta = \frac{T}{T_{S1}} + \frac{T}{T_{S2}} + \frac{T}{T_{S3}}$$
$$= \frac{10}{20} + \frac{10}{30} + \frac{10}{60}$$
$$= 1 \text{ rad}$$

15.

$$P = 5000 \text{ W}$$
$$N = 200 \text{ rpm}$$
$$\mu = 0.25$$
$$r_i = 0.025 \text{ m}$$
$$p = 1 \times 10^6 \text{ Pa}$$

Friction torque is calculated as

$$P = T \times \frac{2\pi N}{60}$$
$$T = 23.8732 \text{ Nm}$$

For uniform contact pressure p,

$$T = \frac{2}{3}\mu \times p\pi \left(r_o^2 - r_i^2\right) \times \frac{\left(r_o^3 - r_i^3\right)}{\left(r_o^2 - r_i^2\right)}$$
$$= \frac{2}{3}\mu p\pi \left(r_o^3 - r_i^3\right)$$

Therefore,

$$r_o = 39.4121 \text{ mm}$$

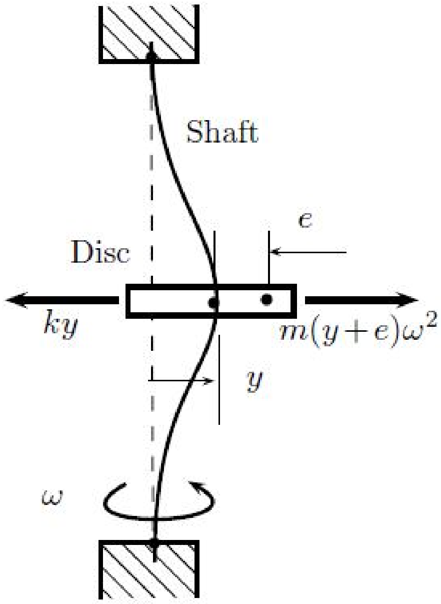
16. Given that

$$\alpha = 19.5^o$$
$$\omega_1 = 500 \text{ rpm}$$

Maximum speed of the driven shaft is $\omega_2 = \omega_1/\cos\alpha$

$= 531.91$ rpm

17. When a rotor is mounted on a shaft, its center of mass does not usually coincide with the center line of the shaft. Therefore, when the shaft rotates, it is subjected to a centrifugal force which makes the shaft to bend in the direction of eccentricity of the center of mass. The shaft tends to bow out at certain speed and whirl in a complicated manner. This increases the eccentricity of the mass, and hence the centrifugal force. In this way, the effect is cumulative and ultimately the shaft can even fail. Critical speed or whirling speed is the speed at which the shaft tends to vibrate violently in transverse direction. This is also called whipping speed.



The unbalanced mass is in equilibrium under the centrifugal force $m(y + e)\omega^2$ and force resisting the deflection key :

$$ky = m(y + e)\omega^2$$

$$y = \frac{e}{(\omega/\omega_n)^2 - 1}$$

18. Notch sensitivity (q) for fatigue loading is defined in terms of actual stress concentration factor kf and the theoretical stress concentration factor kt by the following expression:

$$q = \frac{k_f - 1}{k_t - 1}$$

The value of q is different for different materials and this normally lies between 0 to 0.7. It is small for ductile materials and increases with decrease the ductility.

19. Due to rivet-hole, the maximum stress due to stress concentration is

$$\sigma_{max} = 3\sigma$$

$$= 3 \times 75 = 225 \text{ Mpa}$$

20. ***Creep :*** When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called creep. This property is considered in designing internal combustion engines, boilers and turbines.

Fatigue : When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as fatigue. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.

[PART : B]

21. Given that

$$\sigma_{max} = 250 \text{ MPa}$$
$$\sigma_{min} = 150 \text{ MPa}$$
$$\sigma_{ut} = 450 \text{ MPa}$$
$$\sigma_{yt} = 350 \text{ MPa}$$
$$\sigma_{e} = 250 \text{ MPa}$$

The average and variable stresses are

$$\sigma_a = \frac{250+150}{2} = 200 \text{ MPa}$$

$$\sigma_v = \frac{250-150}{2} = 50 \text{ MPa}$$

Using Soderberg criteria (for ductile materials)

$$\frac{1}{N} = \frac{\sigma_a}{\sigma_{yt}} + \frac{\sigma_v}{\sigma_e} = \frac{200}{350} + \frac{50}{250}$$

$$N = 1.29$$

22. Given that

$$\mu = 30 \times 10^{-3} \text{ Pa·s}$$
$$N = 1800 \text{ rpm}$$
$$p = 2.4 \text{ MPa}$$
$$r/c = 120$$

The speed in rps is determined as

$$N' = \frac{N}{60} = 30 \text{ rps}$$

Sommerfeld number is given by

$$S = \frac{\mu N'}{p}\left(\frac{r}{c}\right)^2 = 5.4 \times 10^{-3}$$

23. **1.** ***Pitch :*** Pitch is the distance between center of one rivet to that of the next adjacent rivet in the same row.

2. Margin Margin or marginal distance is the distance between the edges of the plate to the center line of the adjacent row of rivets.

3. Diagonal pitch Diagonal pitch is the distance between the centers of rivets to the center of the next rivet in the adjacent row measured diagonally.
4. Row Pitch Row pitch is the distance between two adjacent rows of rivets.

24. Given that

$$m = 6 \text{ mm}$$
$$T = 24$$
$$b = 32 \text{ mm}$$
$$\phi = 20°$$
$$P = 3.5 \times 10^3 \text{ W}$$
$$N = 1200 \text{rev/s}$$
$$k_f = 1.5$$
$$Y = 0.3$$
$$d = mT$$
$$= 144 \text{ mm}$$

Pitch circle velocity

$$v = \frac{\pi \times 0.144 \times 1200}{60}$$
$$= 9.04 \text{ m/s}$$

Transmitted load is calculated as

$$F_T = \frac{3.5 \times 10^3}{9.04}$$
$$= 387.17 \text{ N}$$

Using Lewis equation,

$$\sigma = \frac{F_T k_f}{mbY}$$
$$= 10 \text{ MPa}$$

25. Given : $\alpha = 180^o - 160^o = 20^o$; $N = 1500$ r.p.m.; $m = 12$ kg; $k = 100$ mm $= 0.1$ m

We known that angular speed of the driving shaft,

$$\omega = 2\,\pi \times 1500 / 60 = 157 \text{ rad/s}$$

and mass moment of inertia of the driven shaft,

$$I = m.k^2 = 12\,(0.1)^2 = 0.12 \text{ kg - m}^2$$

Maximum angular acceleration of the driven shaft

Let $d\omega_1 / dt$ = Maximum angular acceleration of the driven shaft, and

θ = Angle through which the driving shaft turns.

We know that, for maximum angular acceleration of the driven shaft,

$$\cos 2\theta = \frac{2\sin^2\alpha}{2-\sin^2\alpha} = \frac{2\sin^2 20^\circ}{2-\sin^2 20^\circ} = 0.124$$

$\therefore$ $2\theta = 82.9^\circ$ or $\theta = 41.45^\circ$

and
$$\frac{d\omega_1}{dt} = \frac{\omega^2\cos\alpha.\sin 2\theta.\sin^2\alpha}{(1-\cos^2\theta.\sin^2\alpha)^2}$$

$$= \frac{(157)^2\cos 20^\circ\times\sin 82.9^\circ\times\sin^2 20^\circ}{(1-\cos^2 41.45^\circ\times\sin^2 20^\circ)^2} = 3090 \text{ rad/s}^2 \quad \textbf{Ans.}$$

Maximum torque required

We know that maximum torque required

$$= I\times d\omega_1/dt = 0.12\times 3090 = 371 \text{ N-m} \quad \textbf{Ans.}$$

26. The effort required at the circumference of the screw to lower the load is P = W tan (φ – α) and the torque required to lower the load

$$T = P\times\frac{d}{2} = W\tan(\phi-\alpha)\frac{d}{2}$$

In the above expression, if φ < α, then torque required to lower the load will be negative. In other words, the load will start moving downward without the application of any torque. Such a condition is known as over haulding of screws. If however, φ > α, the torque required to lower the load will positive, indicating that an effort is applied to lower the load. Such a screw is known as self locking screw. In other words, a screw will be self locking if the friction angle is greater than helix angle or coefficient of friction is greater than tangent of helix angle i.e. μ or tan φ > tan α

27. Let a = Distance between the centre lines of the two cylinders.

$\therefore$ Swaying couple

$$= (1-c)m.\omega^2.r\cos\theta\times\frac{a}{2}$$

$$-(1-c)m.\omega^2.r\cos(90^\circ+\theta)\frac{a}{2}$$

$$= (1-c)m.\omega^2.r\times\frac{a}{2}(\cos\theta+\sin\theta)$$

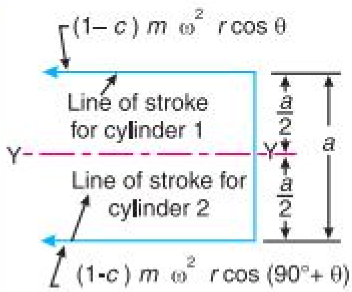


Fig. 22.5. Swaying couple.

The swaying couple is maximum or minimum when $(\cos\theta+\sin\theta)$ is maximum or minimum. For $(\cos\theta+\sin\theta)$ to be maximum or minimum,

$$\frac{d}{d\theta}(\cos\theta+\sin\theta) = 0 \quad \text{or} \quad -\sin\theta+\cos\theta = 0 \quad \text{or} \quad -\sin\theta = -\cos\theta$$

$\therefore$ $\tan\theta = 1$ or $\theta = 45^\circ$ or 225°

Thus, the swaying couple is maximum or minimum when θ = 45° or 225°.

$\therefore$ Maximum and minimum value of the swaying couple

$$= \pm(1-c)m.\omega^2.r\times\frac{a}{2}(\cos 45^\circ+\sin 45^\circ) = \pm\frac{a}{\sqrt{2}}(1-c)m.\omega^2.r$$

28. Given : d = 50 mm = 0.05 m; l = 3 m, W_1 = 1000 N; W_2 = 1500 N; W_3 = 750 N; E = 200 GN/m^2 = 200 × 10^9 N/m^2.

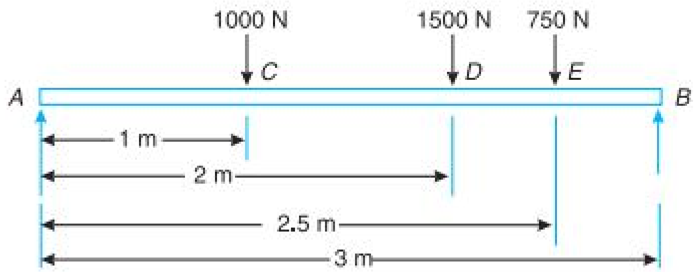
The shaft carrying the loads is shown in figure.

We know that moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4 = 0.307 \times 10^{-6}\,\text{m}^4$$

and the static deflection due to a point load W,

$$\delta = \frac{Wa^2b^2}{3EIl}$$



∴ Static deflection due to a load of 1000 N,

$$\delta_1 = \frac{1000 \times 1^2 \times 2^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 7.24 \times 10^{-3}\ \text{m}$$

. . . (Here a = 1 m, and b = 2 m)

Similarly, static deflection due to a load of 1500 N,

$$\delta_2 = \frac{1500 \times 2^2 \times 1^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 10.86 \times 10^{-3}\ \text{m}$$

. . . (Here a = 2 m, and b = 1 m)

and static deflection due to a load of 750 N,

$$\delta_3 = \frac{750\,(2.5)^2\,(0.5)^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 2.12 \times 10^{-3}\ \text{m}$$

. . . (Here a = 2.5 m, and b = 0.5 m)

We know that frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3}} = \frac{0.4985}{\sqrt{7.24 \times 10^{-3} + 10.86 \times 10^{-3} + 2.12 \times 10^{-3}}}$$

$$= \frac{0.4985}{0.1422} = 3.5\ \text{Hz}\ \textbf{Ans.}$$

29. Thick film lubrication is a state of lubrication in which two surfaces of the bearing are completely separated by a thick film of lubricant. Due to the absence of surface to-surface contact, the viscous resistance arises from the viscosity of the lubricant. The lubricating oil is fed at the point of minimum stress so that it can penetrate inside the bearing.

Thick film lubrication has two regimes of lubrication:

1. Hydrodynamic Lubrication In hydrodynamic lubrication, the load supporting film is maintained by the shape and dynamic motion of the surfaces, such as in engines. The rotating journal climbs the bearing surface and forces the lubricant into the wedge-shaped region between the journal and bearing. The pressure is generated gradually within the system. Due to this feature, these bearings are also called self-acting bearing.

2. Hydrostatic Lubrication Hydrostatic lubrication is achieved by creating the load supporting film by an external source, like a pump. Therefore, these bearings are called externally pressurized bearings. Hydrostatic bearings, although costly, offer advantages, such as high load carrying capacity, no starting friction, no rubbing action.

30.

$$\frac{4F_{st} \times 10^3 \times 1}{\pi 0.04^2 \times 210 \times 10^9} = \frac{4\,(300 - F_{st}) \times 10^3 \times 1}{\pi 0.06^2 \times 70 \times 10^9}$$

$$F_{st} = \frac{4}{3}(300 - F_{st})$$

Also

$$F_{st} + \frac{4}{3}F_{st} = 400$$

$$F_{st} = 171.428 \text{ kN}(-)$$

$$F_{al} = 128.57 \text{ kN}$$

The stresses induced are calculated as

$$\sigma_{st} = -\frac{4 \times 171.428}{\pi \times 0.04^2} = -136.42 \text{ kPa}$$

$$\sigma_{al} = \frac{4 \times 128.57}{\pi \times 0.06^2} = 45.47 \text{ kPa}$$

31. Long or slender structural members loaded axially in compression are called columns. Instead of failing by direct compression, such members can bend and deflect laterally. This type of failure of columns is called buckling. The phenomenon of buckling is explained by the concept of equilibrium:

1. ***Stable Equilibrium :*** When an axial load is applied such that the column remains straight and undergoes only axial compression, the columns is said to be in stable equilibrium, which means that it returns to the straight position.

2. ***Neutral Equilibrium :*** Upon increasing the axial load gradually, the column can bend, and reach a neutral equilibrium where the column can undergo small lateral deflections with no change in the axial force. The corresponding value of the axial load is called the critical load.

3. ***Unstable Equilibrium :*** At the higher values of axial load, the column is in unstable equilibrium where it can collapse by bending.

32.

Let σ_x be the stress in longitudinal direction and $\sigma_y = \sigma_z$ (due to symmetry) the stresses in lateral directions. The stress in longitudinal direction is

$$\sigma_x = -\frac{100 \times 10^3}{50 \times 50}$$
$$= -40 \text{ MPa}$$

Lateral strain in y direction will be given by

$$\varepsilon_y = \frac{1}{E}\{\sigma_y - \mu(\sigma_x + \sigma_z)\}$$

The lateral strains in y or z directions are zero, and $\sigma_y = \sigma_z$, therefore

$$\sigma_y - \mu(\sigma_x + \sigma_z) = 0$$
$$\sigma_y = \sigma_x \frac{\mu}{1-\mu}$$
$$= -17.14 \text{ MPa}$$

The strain in longitudinal is determined as

$$\varepsilon_x = \frac{1}{E}\{\sigma_x - \mu(\sigma_y + \sigma_y)\}$$
$$= 1.4858 \times 10^{-4}$$

Since strain is only in x direction, therefore, the change in volume will be given by

$$\delta V = \varepsilon_x \times V$$
$$= 111.435 \text{ mm}^3$$

[PART : C]

33. Given : m = 2.5 kg; s = 3 N/mm = 3000 N/m; x_6 = 0.25 x_1

We know that natural circular frequency of vibration,

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{3000}{2.5}} = 34.64 \text{ rad/s}$$

Let c = Damping coefficient of the damper in N/m/s,

x_1 = Initial amplitude, and

x_6 = Final amplitude after five consecutive cycles = 0.25 x_1 ...(Given)

We know that

$$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \frac{x_4}{x_5} = \frac{x_5}{x_6}$$

or

$$\frac{x_1}{x_6} = \frac{x_1}{x_2} \times \frac{x_2}{x_3} \times \frac{x_3}{x_4} \times \frac{x_4}{x_5} \times \frac{x_5}{x_6} = \left(\frac{x_1}{x_2}\right)^5$$

$$\therefore \quad \frac{x_1}{x_2} = \left(\frac{x_1}{x_6}\right)^{1/5} = \left(\frac{x_1}{0.25x_1}\right)^{1/5} = (4)^{1/5} = 1.32$$

We know that

$$\log_e\left(\frac{x_1}{x_2}\right) = a \times \frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}}$$

$$\log_e(1.32) = a \times \frac{2\pi}{\sqrt{(34.64)^2 - a^2}} \quad \text{or} \quad 0.2776 = \frac{a \times 2\pi}{\sqrt{1200 - a^2}}$$

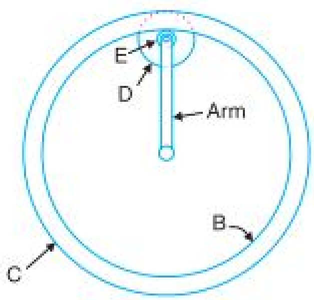
Squaring both sides,

$$0.077 = \frac{39.5\,a^2}{1200 - a^2} \quad \text{or} \quad 92.4 - 0.077\,a^2 = 39.5\,a^2$$

$$\therefore \quad a^2 = 2.335 \quad \text{or} \quad a = 1.53$$

We know that $a = c\,/\,2m$ or $c = a \times 2m = 1.53 \times 2 \times 2.5 = 7.65$ N/m/s

34. Given : $T_B = 80$; $T_C = 82$; $T_D = 28$; $N_A = 500$ r.p.m



First of all, let us find out the number of teeth on wheel E (T_E). Let d_B, d_C, d_D and d_E be the pitch circle diameter of wheels B, C, D and E respectively. From the geometry of the figure,

$$d_B = d_C - (d_D - d_E)$$

or

$$d_E = d_B + d_D - d_C$$

Since the number of teeth are proportional to their pitch circle diameters for the same pitch, therefore

$$T_E = T_B + T_D - T_C = 80 + 28 - 82 = 26$$

The table of motions is given below :

Step No.	*Conditions of motion*	*Revolutions of elements*			
		Arm (or shaft A)	*Wheel B (or shaft F)*	*Compound gear D-E*	*Wheel C*
1.	Arm fixed - wheel B rotated through + 1 revolution (*i.e.* 1 revolution anticlockwise)	0	+ 1	$+\frac{T_B}{T_E}$	$+\frac{T_B}{T_E} \times \frac{T_D}{T_C}$
2.	Arm fixed - wheel B rotated through $+x$ revolutions	0	$+x$	$+x \times \frac{T_B}{T_E}$	$+x \times \frac{T_B}{T_E} \times \frac{T_D}{T_C}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y + x \times \frac{T_B}{T_E}$	$y + x \times \frac{T_B}{T_E} \times \frac{T_D}{T_C}$

Since the wheel C is fixed, therefore from the fourth row of the table,

$$y + x \times \frac{T_B}{T_E} \times \frac{T_D}{T_C} = 0 \quad \text{or} \quad y + x \times \frac{80}{26} \times \frac{28}{82} = 0$$

$\therefore \quad y + 1.05\,x = 0$ ...(*i*)

Also, the shaft A (or the arm) makes 800 r.p.m., therefore from the fourth row of the table,

$y = 800$ ...(*ii*)

From equations (*i*) and (*ii*),

$x = -762$

$\therefore$ Speed of shaft F = Speed of wheel $B = x + y = -762 + 800 = +38$ r.p.m.

= 38 r.p.m. (anticlockwise) **Ans.**

35. Given" N_P = 600 r.p.m. ; $V.R. = T_G/T_P = 4$; σ_{OP} = 84 MPa = 84 N/mm^2 ; σ_{OG} = 105 MPa = 105 N/mm^2 ; T_P = 16 ; m = 8 mm ; b = 90 mm

We know that pitch circle diameter of the pinion,

$$D_P = m.T_P = 8 \times 16 = 128 \text{ mm} = 0.128 \text{ m}$$

$\therefore$ Pitch line velocity,

$$v = \frac{\pi D_P . N_P}{60} = \frac{\pi \times 0.128 \times 600}{60} = 4.02 \text{ m/s}$$

Since the pitch line velocity (v) is less than 12.5 m/s, therefore velocity factor,

$$C_v = \frac{3}{3+v} = \frac{3}{3+4.02} = 0.427$$

We know that for 20° full depth involute teeth, tooth form factor for the pinion,

$$y_P = 0.154 - \frac{0.912}{T_P} = 0.154 - \frac{0.912}{16} = 0.097$$

and tooth form factor for the gear,

$$y_G = 0.154 - \frac{0.912}{T_G} = 0.154 - \frac{0.912}{4 \times 16} = 0.14 \quad \ldots (\because T_G/T_P = 4)$$

$\therefore \quad \sigma_{OP} \times y_P = 84 \times 0.097 = 8.148$

and $\quad \sigma_{OG} \times y_G = 105 \times 0.14 = 14.7$

Since ($\sigma_{OP} \times y_P$) is less than ($\sigma_{OG} \times y_G$), therefore the pinion is weaker. Now using the Lewis equation for the pinion, we have tangential load on the tooth (or beam strength of the tooth),

$$W_T = \sigma_{wP}.b.\pi\, m.y_P = (\sigma_{OP} \times C_v)\, b.\pi\, m.y_P \quad (\because \sigma_{WP} = \sigma_{OP}.C_v)$$
$$= 84 \times 0.427 \times 90 \times \pi \times 8 \times 0.097 = 7870 \text{ N}$$

$\therefore$ Power that can be transmitted

$= W_T \times v = 7870 \times 4.02 = 31\,640$ W = 31.64 kW **Ans.**

36.

1. <u>*Gerber Parabola*</u> A parabolic curve joining σ_e on the ordinate and σ_{ut} on the abscissa is called the *Gerber parabola* of fatigue failure. This criterion fits the failure points of experimental data in the best manner. The following equation presents this failure criterion:

$$\left(\frac{\sigma_a}{\sigma_{ut}}\right)^2 + \frac{\sigma_v}{\sigma_e} = 1$$

To consider a factor of safety N, the above equation is modified as

$$\left(N\frac{\sigma_a}{\sigma_{ut}}\right)^2 + N\frac{\sigma_v}{\sigma_e} = 1$$

2. Soderberg Line A straight line joining σ_e on the ordinate and σ_y on the abscissa is called the *Soderberg line* of fatigue failure. Following is the equation for this failure criterion:

$$\frac{\sigma_a}{\sigma_y} + \frac{\sigma_v}{\sigma_e} = 1$$

To consider a factor of safety N, the equation takes the following form:

$$\frac{\sigma_a}{\sigma_y} + \frac{\sigma_v}{\sigma_e} = \frac{1}{N}$$

This indicates that the factor of safety shifts the criterion's line towards origin.

3. Goodman Line A straight line joining σ_e on the ordinate and σ_{ut} on the abscissa is called the *Goodman line* of fatigue failure. This is represented by the following equation:

$$\frac{\sigma_a}{\sigma_{ut}} + \frac{\sigma_v}{\sigma_e} = 1$$

If a factor of safety N is used, then

$$\frac{\sigma_a}{\sigma_{ut}} + \frac{\sigma_v}{\sigma_e} = \frac{1}{N}$$

Goodman line is a safer option for design consideration because it is completely inside the Gerber parabola. Soderberg line is a more conservative failure criterion.

Figure 5.13 shows the modified criteria of Goodman lines for fluctuating axial or bending stresses (σ) and fluctuating torsional shear stresses (τ).

37. Given : N = 900 r.p.m. or $\omega = 2\pi \times 900 / 60 = 94.26$ rad/s; $\omega_1 - \omega_2 = 2\%\ \omega$ or

$\frac{\omega_1 - \omega_2}{\omega} = C_S = 2\% = 0.02$; D = 650 mm or R = 325 mm = 0.325 m ; $\rho = 7200$ kg / m^3

Mass of the flywheel rim

Let m = Mass of the flywheel rim in kg.

First of all, let us find the maximum fluctuation of energy. The turning moment diagram for a multi-cylinder engine is shown in Fig. 22.7.

Since the scale of turning moment is 1 mm = 70 N-m and scale of the crank angle is 1 mm = 4.5° = π / 40 rad, therefore 1 mm^2 on the turning moment diagram.

$= 70 \times \pi / 40 = 5.5$ N-m

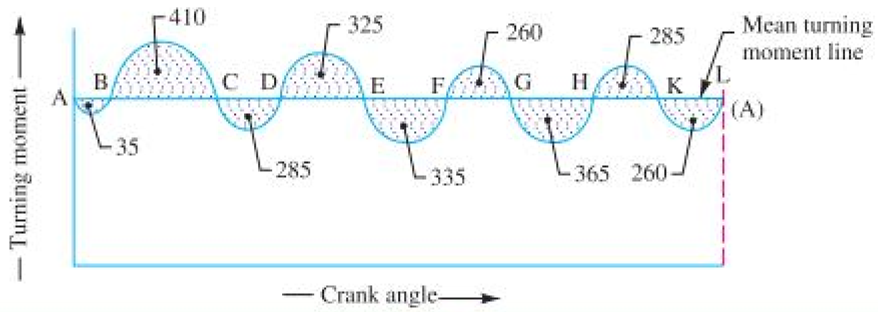


Fig. 22.7

Let the total energy at $A = E$. Therefore from Fig. 22.7, we find that

Energy at B = $E - 35$

Energy at C = $E - 35 + 410 = E + 375$

Energy at D = $E + 375 - 285 = E + 90$

Energy at E = $E + 90 + 325 = E + 415$

Energy at F = $E + 415 - 335 = E + 80$

Energy at G = $E + 80 + 260 = E + 340$

Energy at H = $E + 340 - 365 = E - 25$

Energy at K = $E - 25 + 285 = E + 260$

Energy at L = $E + 260 - 260 = E$ = Energy at A

From above, we see that the energy is maximum at E and minimum at B.

$\therefore$ Maximum energy $= E + 415$

and minimum energy $= E - 35$

We know that maximum fluctuation of energy,

$= (E + 415) - (E - 35) = 450$ mm^2

$= 450 \times 5.5 = 2475$ N-m

We also know that maximum fluctuation of energy (ΔE),

$2475 = m.R^2.\omega^2.C_S = m\,(0.325)^2\,(94.26)^2\,0.02 = 18.77\,m$

$\therefore$ $m = 2475 / 18.77 = 132$ kg **Ans.**

Cross-section of the flywheel rim

Let t = Thickness of the rim in metres, and

b = Width of the rim in metres = 2 t ...(Given)

$\therefore$ Area of cross-section of the rim,

$A = b \times t = 2\,t \times t = 2\,t^2$

We know that mass of the flywheel rim (m),

$132 = A \times 2\,\pi\,R \times \rho = 2\,t^2 \times 2\,\pi \times 0.325 \times 7200 = 29\,409\,t^2$

$\therefore$ $t^2 = 132 / 29\,409 = 0.0044$ or $t = 0.067$ m = 67 mm **Ans.**

and $b = 2t = 2 \times 67 = 134$ mm **Ans.**

38. The cross-sectional areas of the steel and bronze rods are 500 mm2 and 200 mm2, respectively. Moduli of elasticity of their material are 180 GPa and 80 GPa, respectively. Calculate the stresses in the steel and bronze rods.

Let F_s and F_b are the loads shared by the steel and bronze rods respectively. As the rod is rigid, therefore, the deformation of steel and bronze rods are related as

$$\frac{\delta_s}{0.5} = \frac{\delta_b}{1.5}$$

$$\frac{F_s \times 1.0}{500 \times 180} = \frac{0.5}{1.5} \times \frac{F_b \times 1.0}{200 \times 80}$$

$$F_s = 1.875 F_b$$

For equilibrium, taking moments about hinged point A,

$$40 \times 2.5 = F_s \times 0.5 + F_b \times 1.5$$

$$F_b = \frac{40 \times 2.5}{0.5 \times 1.875 + 1.5}$$

$$= 41.02 \text{ kN}$$

$$F_s = 1.875 F_b$$

$$= 76.923 \text{ kN}$$

The stresses in steel and bronze rods are given by

$$\sigma_s = \frac{76.923 \times 10^3}{500 \times 10^{-6}}$$

$$= 153.85 \text{ MPa}$$

$$\sigma_b = \frac{41.02 \times 10^3}{200 \times 10^{-6}}$$

$$= 205.06 \text{ MPa}$$

39. The moment of inertia of the section is

$$I = \frac{120 \times 180^3}{12}$$

$$= 58.32 \times 10^6 \text{ mm}^4$$

The shear force at the section 1 m from left end is

$$V = \frac{4 \times 6}{2} - 4 \times 1$$

$$= 8 \text{ kN}$$

To calculated the shear stress at layer 30 mm from top of the section, area above the layers is $A = 120 \times 30 \text{ mm}^2$ and its centroid is situated at $\bar{y} = 75$ mm from the NA of the beam. Therefore

$$A'\bar{y} = 120 \times 30 \times 75$$
$$= 270 \times 10^3 \text{ mm}^3$$

Therefore, the maximum shear stress is

$$\tau = \frac{V}{Ib} A'\bar{y}$$
$$= \frac{8}{58.32 \times 120} \times 270$$
$$= 0.3086 \text{ N/mm}^2$$
$$= 308.6 \text{ kPa}$$

Maximum value of shear force at the ends, given by

$$V = \frac{4 \times 6}{2}$$
$$= 12 \text{ kN.}$$

The maximum shear stress occurs at the top lamina of the section, given by

$$\tau = 1.5 \frac{V}{bh}$$
$$= 833.33 \text{ kPa}$$

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